ప్రతిభ

TECHNIQUE OF SOLVING PROBLEMS

Preparation for IIT(JEE), should not be memory oriented, but should be concept oriented. A student preparing for other entrances, generally concentrates on principles, concepts and derivations of various formulae. In addition to these things a serious aspirant of IIT(JEE) should also concentrate on limitations and applications of each and every formula. One should also practice in doing a given problem by different methods. Any aspirant should not neglect even minor fraction of syllabus. As far as I.I.T (JEE) is concerned , a student should mostly depend upon more logic-based problems rather than memory - based.

To be perfect in solving problem is more important than doing huge number of problems. Perfection in the intermediate syllabus, followed by practicing complicated problems based on the simple basics is the correct modus operandi for achieving success in the JEE. Understanding the concept(s) behind a problem is important but more important is the method and technique of solving it. Here under are a few typical examples. Observe carefully the techniques adopted to solve the problem.

IIT - MATHS


D.KRISHNAMURTHY

1. If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal then a, b, c are in

A) A.P. B) G.P. C) H.P. D) None of these

(Ans: C)

Sol: The normal method of solving it is, equating the discriminant to zero. But simplification takes time. The following method is better.

By inspection, x = 1 is a root. So the other root is also 1.

$\therefore$ Sum of roots $= \frac{-b(c-a)}{a(b-c)} = 1+1 = 2$

$\Rightarrow b = {}^{2ac}/_{a+c}$ $\therefore$ a, b, c are in H.P.

2. The equation of the line, equidistant from A(1, 3), B(3, 5) and making equal (non zero) intercepts with co-ordinate axes is __ ***(Ans: x + y - 6 = 0)***

Sol: Every line passing through the midpoint of AB is equidistant from A and B. Let the required line be x + y = a. As it passes through (2, 4), a = 6.

$\therefore$ x + y = 6 is the line.

3. If θ is acute, the value of

$$\frac{2\sec\theta+3\tan\theta+5\sin\theta-7\cos\theta+5}{2\tan\theta+3\sec\theta+5\cos\theta+7\sin\theta+8}$$ **is**

A) $\frac{1-\cos\theta}{\sin\theta}$ B) $\frac{\sin\theta}{1+\cos\theta}$

C) $\frac{\tan\theta}{\sec\theta+1}$ D) $\frac{\sec\theta-1}{\tan\theta}$

(Ans: A, B, C, D)

Sol: One can easily observe that,

$$\frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta} = \frac{\tan\theta}{\sec\theta+1} = \frac{\sec\theta-1}{\tan\theta}$$

$$\Rightarrow \frac{7(1-\cos\theta)}{7\sin\theta} = \frac{5\sin\theta}{5(1+\cos\theta)} = \frac{3\tan\theta}{3(\sec\theta+1)} = \frac{2(\sec\theta-1)}{2\tan\theta}$$

Each of these fractions is equal to (sum of numerators)/(sum of denominators).

4. The number of ways of arranging 6 boys and 6 girls in a row such that two extreme positions are occupied by boys and between any two boys an even number of girls sit is__. *(Ans: 5<6<7)*

Sol: If the boys are arranges as $B_1 \times B_2 \times B_3 \times B_4 \times B_5 \times B_6$ there are 5 places for girls, in which we can place even number of girls. If we assume that all girls are alike, then number of ways of placing girls is, number of non negative integral solutions of $2x_1 + 2x_2 + 2x_3 + 2x_4 + 2x_5 = 6 \Rightarrow x_1 + x_2 + ... + x_5 = 3$ which is ${}^{(3+5-1)}C_{5-1} = {}^7C_4$. Since the girls are different number of always is ${}^7C_4$ x < 6. Boys can be rearranged in <6 ways.

$\therefore$ Required number of arrangements is

${}^7C_4$ x < 6 x < 6

$= {}^{<7}/_{<4.\ <3}$ < 6 < 6 = < 5< 6 < 7

5. The equation of the largest circle having center at (1, 0) and inscribed in the ellipse $x^2 + 4y^2 = 16$ is _____.

(Ans: $(x + 1)^2 + y^2 = {}^{33}/_9$)

Hint: Normal to the ellipse at 'θ' passes through (1, 0).

6. Find the equation to the smallest circle passing through the points of intersection of the curves given by $y^2(x^2 + y^2 - x - 2) - x(x^2 - 2) = 0$.

(Ans: $x^2 + y^2 - 2x = 0$)

Sol: Given equation can be written as $(x^2 + y^2 - 2)(y^2 - x) = 0$ the curves are $x^2 + y^2 = 2$ and $y^2 = x$. Solve to get the points of intersection P and Q. Required circle will have PQ as diameter.

7. Evaluate $\int \frac{\cos 5x + \cos 4x}{1-2\cos 3x}$

(Ans: $- \sin x - {}^1/_2 \sin 2x + c$)

Hint: Multiply numerator and denominator by sin 3x and apply transformations formulae.

8. Evaluate $\int \frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{(x\sqrt{x}+x+\sqrt{x})} dx$

(Ans: ${}^{x^2}/_2 - x + c$)

Hint: Multiply Numerator and Denominator by $(\sqrt{x} - 1)$.

9. If $\int_1^2 e^{x^2} dx = K$ **find** $\int_e^{e^4} \sqrt{\log x dx}$

(Ans: $2e^4 - e - K$).

Sol: Put $\log x = t^2$ to get

$$\int_e^{e^4} \sqrt{\log x dx} = \int_1^2 2t^2.e^{t^2} dt$$

$\int_1^2 t.(e^{t^2} 2t)\, dt$. Integrate by parts.